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Sublinear-Time Language Recognition and Decision by One-Dimensional Cellular Automata

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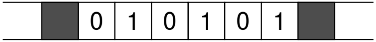
Cellular Automata

- Here as language acceptors
- “Usual” acceptance condition: leftmost active cell is accepting

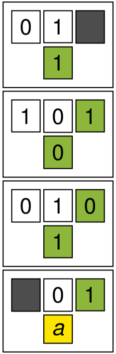
Step 0



Cellular Automata

- Here as language acceptors
- "Usual" acceptance condition: leftmost active cell is accepting

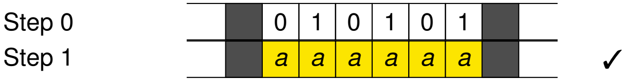
Step 0		0	1	0	1	0	1	
Step 1		0	1	0	1	0	1	
Step 2		0	1	0	1	0	1	
Step 3		0	1	0	1	0	1	
Step 4		0	1	0	1	0	1	
Step 5		0	1	0	1	0	1	
Step 6		*a*	1	0	1	0	1	



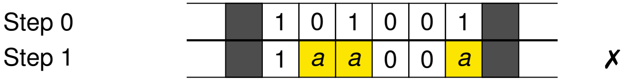
- Accepts $\{01\}^+$ in n steps (where n is the input length)
- Sublinear-time languages are trivial

ACA

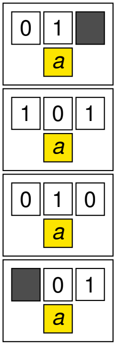
- Acceptance condition: *all* cells accept *simultaneously*



- Accepts $\{01\}^+$ in 1 step
- Example for not accepting:



- ACAs are capable of non-trivial sublinear-time computation



(Strictly) Locally Testable

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- $L \subseteq \Sigma^*$ is *strictly locally testable* (SLT) if:
  - There is $k \in \mathbb{N}_+$ as well as sets $\pi, \sigma \subseteq \Sigma^{\le k}$ and $\mu \subseteq \Sigma^k$
  - such that $w \in L$ iff
    - k-prefix of w is in π
    - every k-infix of w is in μ
    - k-suffix of w is in σ
- $L \subseteq \Sigma^*$ is *locally testable* (LT) if:
  - For all $w_1, w_2 \in \Sigma^*$ such that:
    - w_1 and w_2 have the same k-prefix
    - w_1 and w_2 have the same set of k-infixes
    - w_1 and w_2 have the same k-suffix
  - We have: $w_1 \in L$ iff $w_2 \in L$
- SLT $\subsetneq$ LT $\subsetneq$ REG

Previous Work

- $\mathsf{ACA}(t)$: class of languages acceptable by an ACA in time $\le t$
- Sommerhalder and Westrhenen (1983):

  $\mathsf{ACA}(O(1)) = \mathsf{SLT}_\vee$

  where $\mathsf{SLT}_\vee$ is closure of SLT under union
- Ibarra, Palis, and S. M. Kim (1985):

  $\mathsf{ACA}(o(\log n)) \subseteq \mathsf{REG}$

  and this bound is *tight*
- S. Kim and McCloskey (1990) consider a slightly different acceptance condition
  - Constant-time class of resulting model equals SLT

Our Contributions

We prove several results regarding the classes $\mathsf{ACA}(t)$ for sublinear t:

- Time hierarchy for “most” $t \in \omega(\log n) \cap O(n)$
- Intersection with REG
  and an improvement on Ibarra, Palis, and S. M. Kim’s result
- Inclusion in SC and AC

We also consider a *decider* version of ACA (DACA)

Result 1: Time Hierarchy

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- For every “nice” $t \in \omega(\log n) \cap O(n)$:

  $\mathrm{ACA}(o(t)) \subsetneq \mathrm{ACA}(t)$

- For example, $t \in \Theta(\log n)^a$ and $t \in \Theta(n^{1/b})$ are all “nice”
  - where $a > 1$ and $b \geq 1$
- What about a hierarchy in $\mathrm{ACA}(o(\log n))$?

Result 2: Intersection with REG

- Improvement to Ibarra, Palis, and S. M. Kim (1985):

  $\mathsf{ACA}(o(\log n)) = \mathsf{ACA}(O(1))$

  and this bound is *tight*

- $\mathsf{ACA}(t)$ does not seem to "grow" with respect to REG:

  $\mathsf{ACA}(o(n)) \cap \mathsf{REG} \subsetneq \mathsf{LT}$

- Open question: Is $\mathsf{ACA}(o(n)) \cap \mathsf{REG} \subsetneq \mathsf{SLT}_\vee = \mathsf{ACA}(O(1))$?

Result 3: Relation to SC and AC

- Definitions:
  - SC^k: problems decidable by a TM in $O(\log n)^k$ space and poly(n) time
  - AC^k: problems decidable by a unbounded fan-in circuit of $O(\log n)^k$ depth and poly(n) size
- The following inclusions hold for every $k \in \mathbb{N}_+$:

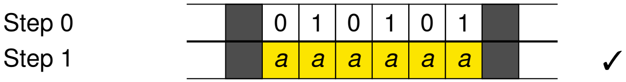
$$\mathsf{ACA}(O(\log n)^k) \subsetneq \mathsf{SC}^k$$

$$\mathsf{ACA}(O(\log n)^k) \subsetneq \text{L-uniform-}\mathsf{AC}^{k-1}$$

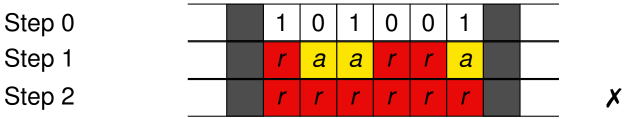
- (Known: $\mathsf{SC}^1 \subseteq \mathsf{AC}^1$ and $\mathsf{SC}^1 \not\subseteq \mathsf{AC}^0$)
- Small caveat: usual uniformity condition for AC^0 is (deterministic) logarithmic *time*
  - For AC^0, we can replace $O(\log n)$ space above with $O(\log n)^2$ time
  - Open question: Does inclusion also hold for $O(\log n)$ time uniformity?

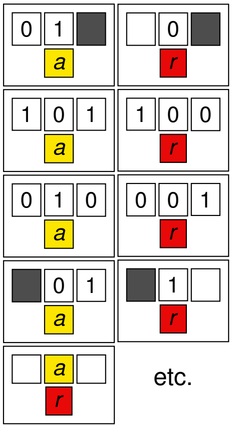
DACA

- Decider version of ACA
- Additional condition: inputs not accepted must be *explicitly rejected*
- Rejection condition: all cells are rejecting simultaneously



- Still accepts $\{01\}^+$ in 1 step
- Example for *rejecting*:





DACA: Results

- For constant-time DACA:

  $\mathrm{DACA}(O(1)) = \mathrm{LT}$

  This implies $\mathrm{ACA}(O(1)) \subsetneq \mathrm{DACA}(O(1))$!

- For sublinear-time DACA:

  $\mathrm{DACA}(o(\sqrt{n})) = \mathrm{DACA}(O(1))$

  and this bound is *tight*

- Open problems:
  - Does time hierarchy extend to DACAs?
  - For which t is $\mathrm{DACA}(t)$ closed under union or intersection?
    - $\mathrm{DACA}(O(1)) = \mathrm{LT}$ is closed under union and intersection

References I

[1] Oscar H. Ibarra, Michael A. Palis, and Sam M. Kim. “Fast Parallel Language Recognition by Cellular Automata.” In: *Theor. Comput. Sci.* 41 (1985), pp. 231–246.

[2] Sam Kim and Robert McCloskey. “A Characterization of Constant-Time Cellular Automata Computation.” In: *Phys. D* 45.1-3 (Oct. 1990), pp. 404–419.

[3] Rudolph Sommerhalder and S. Christian van Westrhenen. “Parallel Language Recognition in Constant Time by Cellular Automata.” In: *Acta Inf.* 19 (1983), pp. 397–407.