Compositions of Constant Weighted Extended Tree Transducers

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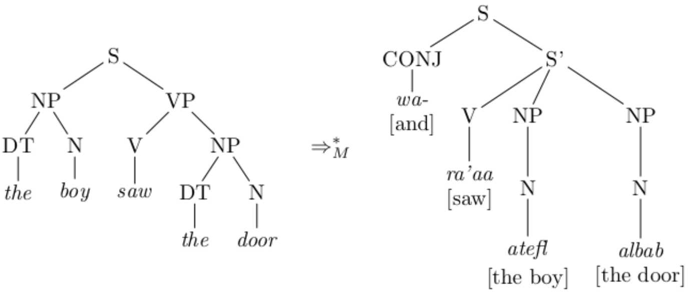


Figure: English-to-Arabic translation [2].

Let $X = \{x_i \mid i \in \mathbb{N}\}$ a set of (formal) variables.

Ranked alphabet

A ranked alphabet is a tuple (Σ, rk), where

- Σ is a finite set
- $\mathrm{rk} : \Sigma \to \mathbb{N}$ is a function

Trees

The set $T_\Sigma(A)$ of all trees is the smallest set T such that

- $A \subseteq T$
- $\sigma(t_1, \ldots, t_k) \in T \ \forall \sigma \in \Sigma^{(k)}$, and $t_1, \ldots, t_k \in T$

Instead of $T_\Sigma(\emptyset)$ we simply write T_Σ.

Semiring

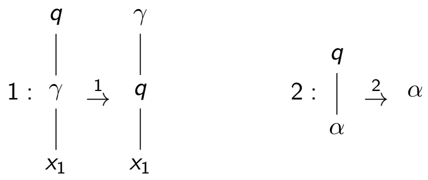
A commutative semiring is an algebraic structure $(S, +, \cdot, 0, 1)$, such that

- $(S, +, 0)$,$(S, \cdot, 1)$ are commutative monoids
- $\cdot$ distributes over $+$
- $s \cdot 0 = 0$ for all $s \in S$

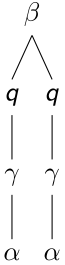
A semiring is *idempotent*, if $1 + 1 = 1$.
$(\{0, 1\}, \max, \min, 0, 1)$ is called the BOOLEAN semiring.

Let $(S, +, \cdot, 0, 1)$ be an arbitrary commutative semiring.

Let $M = (\{q\}, \Sigma, \Delta, \{q\}, R)$ be a wtdtt with $\Sigma = \Delta = \{\beta^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and



Behaviour:



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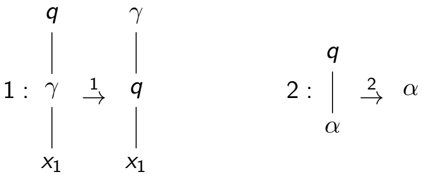

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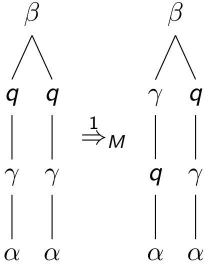

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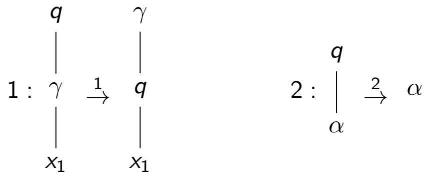

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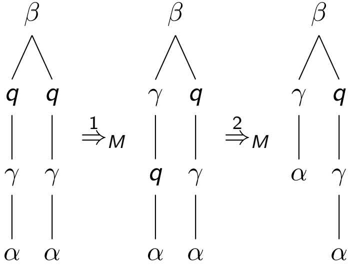

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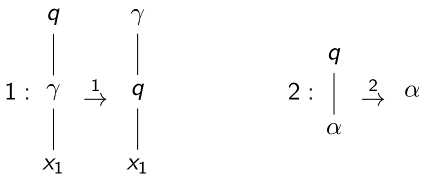

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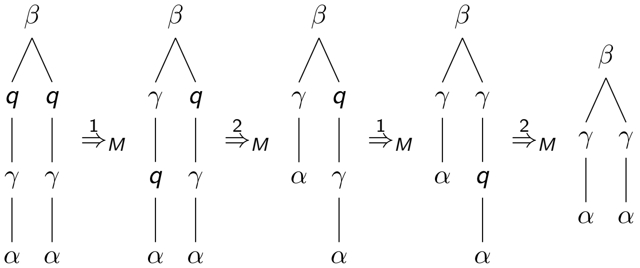

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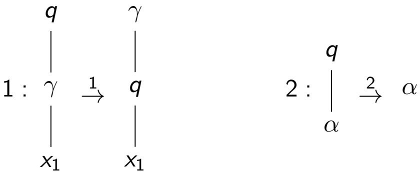

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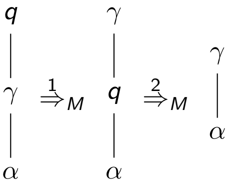

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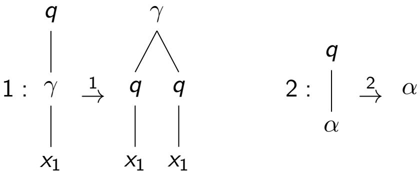

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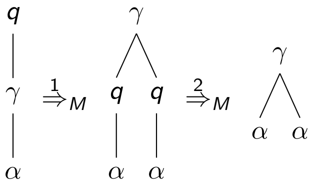


- $D_M^{\perp}(t)$: Set of all *terminal derivations*

- $D^{\perp}_{M}(t)$: Set of all *terminal derivations*

Weighted Relation modeled by a wxtt

M models the *weighted relation* $\tau_M \colon T_\Sigma \times T_\Delta \to S$ given by

$$\tau_M(t, u) = \sum_{q \in Q_0} \Big(\sum_{\substack{d \in D^{\perp}_{M}(q(t)) \\ d(q(t)) = u}} \mathit{weight}(d) \Big)$$

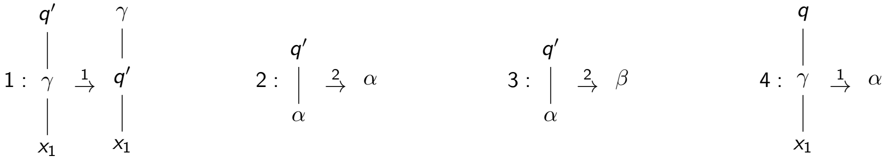
Let M', M two wxtt's with $\tau_{M'} \colon T_\Sigma \times T_\Delta \to S$, $\tau_M \colon T_\Delta \times T_\Gamma \to S$

Composition of wxtt's

The composition $(\tau_{M'} \, ; \tau_M) \colon T_\Sigma \times T_\Gamma \to S$ is given by

$$(\tau_{M'} \, ; \tau_M)(t, t'') = \sum_{t' \in T_\Delta} \tau_{M'}(t, t') \cdot \tau_M(t', t'')$$

Let M', M wxtt's with:



Let M', M wxtt's with:

$$1: \; q'(\gamma(x_1)) \xrightarrow{1} \gamma(q'(x_1)) \qquad 2: \; q'(\alpha) \xrightarrow{2} \alpha \qquad 3: \; q'(\alpha) \xrightarrow{2} \beta \qquad 4: \; q(\gamma(x_1)) \xrightarrow{1} \alpha$$

$$t = \underbrace{\gamma\Big(\cdots\gamma}_{n \text{ times } \gamma}(\alpha)\cdots\Big)$$

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$$t = \underbrace{\gamma\big(\cdots\gamma}_{n \text{ times } \gamma}(\alpha)\cdots\big) \qquad \text{we have} \qquad D^{\perp}_{M'}\big(q'(t)\big) = \big\{\underbrace{1\cdots1}_{n \text{ times}}2, \underbrace{1\cdots1}_{n \text{ times}}3\big\}$$

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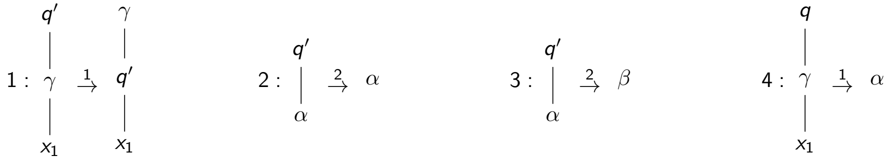

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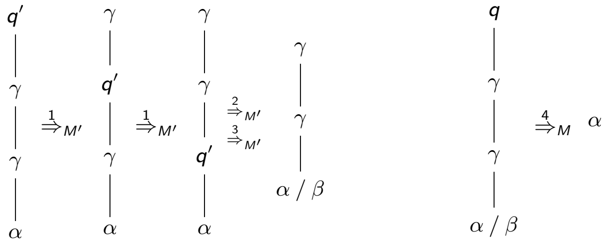
Illustration of $(\tau_{M'}; \tau_M)(\gamma(\gamma(\alpha)), \alpha)$:



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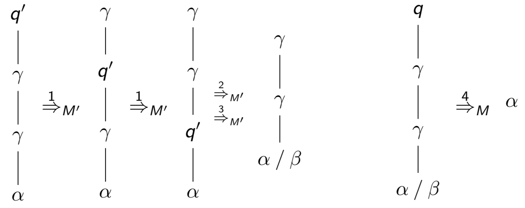
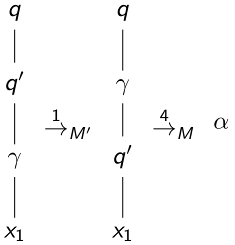


Rule of our Composition construction:



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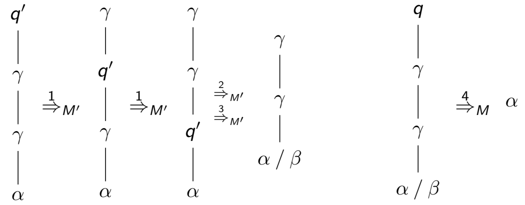

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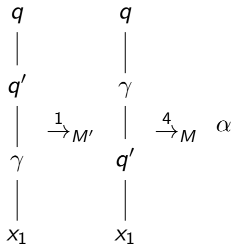

With weight $s_1 \cdot s_4 \cdot 4 = (\tau_{M'},\tau_M)(\gamma(\gamma(\alpha)),\alpha)$.

Definition (Constant see [1, Example 10])

A state $q \in Q$ is *c-constant* for a $c \in S$, if

$$c = \sum_{d \in D^{\perp}_{M}(q(t))} \mathit{weight}(d)$$

for every $t \in T_\Sigma$. The wxtt M is *constant* if for every state $q \in Q$ there exists $c_q \in S$ such that q is c_q-constant.

The Main Result

Theorem

Let M' be a constant wxtt and M be a linear wtdtt. Then there exists a wxtt N such that $\tau_N = \tau_{M'} ; \tau_M$.

Lemma

1-constantness is decidable for BOOLEAN *wxtt over idempotent semirings.*

Corollary

1-constantness is decidable for wxtt over the BOOLEAN *semiring.*

Bibliography

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